B SOLUTION BY FACTORISATION

For quadratic equations which are not of the form $x^2 = k$, we need an alternative method of solution. One method is to **factorise** the quadratic and then apply the **Null Factor law**.

The **Null Factor law** states that:

When the product of two or more numbers is zero, then *at least one* of them must be zero.

So, if $\boldsymbol{ab = 0}$ then $\boldsymbol{a = 0}$ or $\boldsymbol{b = 0}$.

To solve quadratic equations by factorisation, we follow these steps:

Step 1: If necessary, rearrange the equation so one side is **zero**.
Step 2: **Fully factorise** the other side (usually the LHS).
Step 3: Apply the **Null Factor law**.
Step 4: **Solve** the resulting linear equations.

Example 3 **Self Tutor**

Solve for x: $x^2 = 3x$

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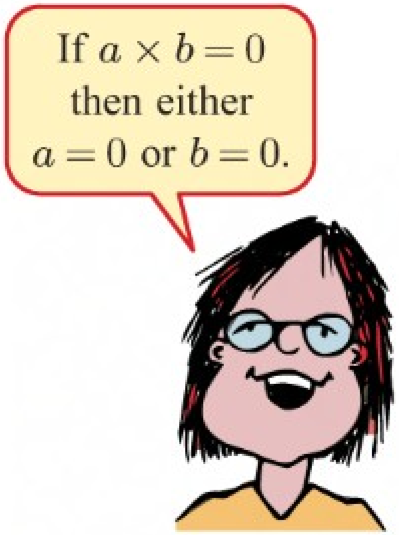
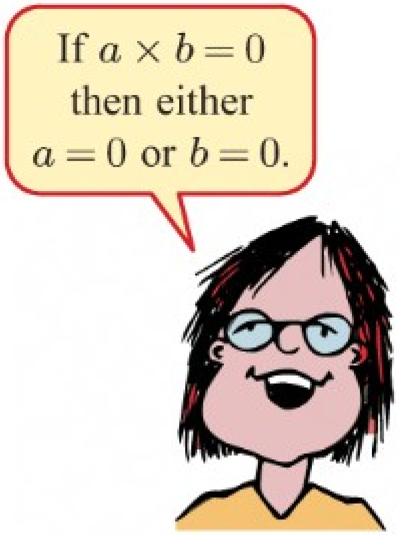

If $a \times b = 0$ then either $a = 0$ or $b = 0$.

Example 3 Self Tutor

Solve for x: $x^2 = 3x$

$$\begin{aligned} x^2 &= 3x \\ \therefore\ x^2 - 3x &= 0 && \{\text{making the RHS} = 0\} \\ \therefore\ x(x-3) &= 0 && \{\text{factorising the LHS}\} \\ \therefore\ x = 0 \text{ or } x - 3 &= 0 && \{\text{Null Factor law}\} \\ \therefore\ x &= 0 \text{ or } 3 \end{aligned}$$


If $a \times b = 0$ then either $a = 0$ or $b = 0$.

WARNING ON INCORRECT CANCELLING

Let us reconsider the equation $x^2 = 3x$ from **Example 3**.

We notice that there is a common factor of x on both sides.

If we cancel x from both sides, we will have $\frac{x^2}{x} = \frac{3x}{x}$ and thus $x = 3$.

Consequently, we will 'lose' the solution $x = 0$.

From this example we conclude that:

We must never cancel a variable that is a common factor from both sides of an equation unless we know that the factor cannot be zero.

Example 4 **Self Tutor**

Solve for x: $x^2 + 3x = 28$

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Solve for x: $x^2 + 3x = 28$

$$\begin{aligned} x^2 + 3x &= 28 \\ \therefore\ x^2 + 3x - 28 &= 0 && \{\text{making the RHS} = 0\} \\ \therefore\ (x+7)(x-4) &= 0 && \{\text{the numbers } +7 \text{ and } -4 \text{ have sum } 3 \text{ and product } -28\} \\ \therefore\ x + 7 = 0 \ \text{ or } \ x - 4 &= 0 && \{\text{Null Factor law}\} \\ \therefore\ x &= -7 \ \text{ or } \ 4 \end{aligned}$$

Check: If $x = -7$, then $x^2 + 3x = (-7)^2 + 3(-7) = 49 - 21 = 28$ ✓

If $x = 4$, then $x^2 + 3x = 4^2 + 3(4) = 16 + 12 = 28$ ✓

Example 5 **Self Tutor**

Solve for x: $5x^2 = 3x + 2$

Example 5 **Self Tutor**

Solve for x: $5x^2 = 3x + 2$

$$5x^2 = 3x + 2$$

$$\therefore\ 5x^2 - 3x - 2 = 0 \qquad \{\text{making the RHS} = 0\}$$

We need two numbers with sum -3 and product -10. These are -5 and $+2$.

$$\therefore\ 5x^2 - 5x + 2x - 2 = 0 \qquad \{\text{`splitting' the middle term}\}$$

$$\therefore\ 5x(x-1) + 2(x-1) = 0$$

$$\therefore\ (x-1)(5x+2) = 0$$

$$\therefore\ x - 1 = 0 \text{ or } 5x + 2 = 0 \qquad \{\text{Null Factor law}\}$$

$$\therefore\ x = 1 \text{ or } -\tfrac{2}{5}$$

1 Solve for x:

a $x^2 - 7x = 0$
b $x^2 - 5x = 0$
c $x^2 = 8x$
d $x^2 = 4x$
e $3x^2 + 6x = 0$
f $2x^2 + 5x = 0$
g $4x^2 - 3x = 0$
h $4x^2 = 5x$
i $3x^2 = 9x$

2 Solve for x:

a $x^2 + 3x + 2 = 0$
b $x^2 - 3x + 2 = 0$
c $x^2 - 10x + 25 = 0$
d $x^2 + 5x + 6 = 0$
e $x^2 + 6 = 5x$
f $x^2 + 7x = -6$
g $x^2 + 14 = -9x$
h $x^2 + 11x = -30$
i $x^2 = 15 - 2x$
j $x^2 + 4x = 12$
k $x^2 = 11x - 24$
l $x^2 = 14x - 49$

3 Solve for x:

a $2x^2 + 18x + 36 = 0$
b $-x^2 - 11x - 28 = 0$
c $3x^2 + 6x = 24$
d $2x^2 + 2x = 24$
e $4x^2 + 24 = 20x$
f $5x^2 + 20 = 20x$
g $3x^2 = 3x + 18$
h $-x^2 = 7x - 60$
i $140 + 6x = 2x^2$
j $2x^2 - 5x + 2 = 0$
k $3x^2 + 8x - 3 = 0$
l $3x^2 + 17x + 20 = 0$
m $2x^2 + 5x = 3$
n $2x^2 + 5 = 11x$
o $2x^2 + 7x + 5 = 0$

4 Solve for x:

a $3x^2 + 13x + 4 = 0$
b $5x^2 - 6 = 13x$
c $2x^2 + 17x = 9$
d $2x^2 = 3x + 5$
e $3x^2 = 8 - 2x$
f $2x^2 = 18 - 9x$
g $-6x^2 + 17x + 3 = 0$
h $-2x^2 - 5x + 12 = 0$
i $12x^2 - 22x + 6 = 0$
j $9x^2 + 6x - 48 = 0$
k $28x^2 - 8 = 2x$
l $36x^2 + 39x = 12$

5 Solve for x:

a $x(x+5) + 2(x+6) = 0$
b $x(1+x) + x = 3$
c $(x-1)(x+9) = 8x$
d $3x(x+2) - 5(x-3) = 17$
e $4x(x+1) = -1$
f $2x(x-6) = x - 20$
g $x(8-x) + 20 = -13$
h $(8x+1)(x-2) = 13 - x$

Example 6 **Self Tutor**

Solve for x: $\dfrac{x-2}{x} = \dfrac{6+x}{2}$

Example 6

Self Tutor

Solve for x: $\dfrac{x-2}{x} = \dfrac{6+x}{2}$

$$\frac{x-2}{x} = \frac{6+x}{2} \qquad \{\text{LCD} = 2x\}$$

$$\therefore \frac{2 \times (x-2)}{2 \times x} = \frac{x \times (6+x)}{x \times 2} \qquad \{\text{to achieve a common denominator}\}$$

$$\therefore 2(x-2) = x(6+x) \qquad \{\text{equating numerators}\}$$

$$\therefore 2x - 4 = 6x + x^2$$

$$\therefore x^2 + 4x + 4 = 0$$

$$\therefore (x+2)^2 = 0$$

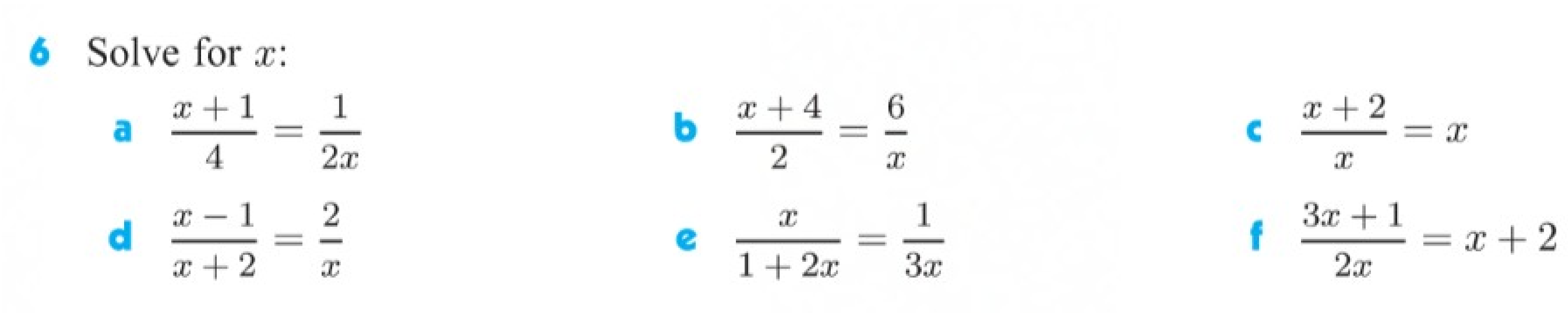
$$\therefore x + 2 = 0$$

$$\therefore x = -2$$

Check: If $x = -2$ then

$$\text{LHS} = \frac{(-2)-2}{(-2)} = \tfrac{-4}{-2} = 2$$

and $$\text{RHS} = \frac{6+(-2)}{2} = \tfrac{4}{2} = 2 \quad ✓$$



6 Solve for x:

a $\dfrac{x+1}{4} = \dfrac{1}{2x}$

b $\dfrac{x+4}{2} = \dfrac{6}{x}$

c $\dfrac{x+2}{x} = x$

d $\dfrac{x-1}{x+2} = \dfrac{2}{x}$

e $\dfrac{x}{1+2x} = \dfrac{1}{3x}$

f $\dfrac{3x+1}{2x} = x+2$

Example 7 **Self Tutor**

Solve for x: $\quad \dfrac{1}{x} + \dfrac{4}{x+6} = 1$

Example 7 **Self Tutor**

Solve for x: $\dfrac{1}{x} + \dfrac{4}{x+6} = 1$

$$\frac{1}{x} + \frac{4}{x+6} = \frac{1}{1} \qquad \{\text{LCD} = x(x+6)\}$$

$$\therefore \quad \frac{1 \times (x+6)}{x \times (x+6)} + \frac{4 \times x}{(x+6) \times x} = \frac{1 \times x(x+6)}{1 \times x(x+6)} \qquad \{\text{to achieve a common denominator}\}$$

$$\therefore \quad x + 6 + 4x = x(x+6) \qquad \{\text{equating numerators}\}$$

$$\therefore \quad 5x + 6 = x^2 + 6x$$

$$\therefore \quad 0 = x^2 + x - 6$$

$$\therefore \quad (x+3)(x-2) = 0$$

$$\therefore \quad x = -3 \ \text{ or } \ 2$$

7 Solve for x:

a $\dfrac{2}{x}+\dfrac{3}{x+2}=1$

b $\dfrac{5}{x}+\dfrac{x}{x-6}=2$

c $\dfrac{4}{x+1}+\dfrac{3}{x+3}=-1$

d $\dfrac{5}{x+2}-\dfrac{6}{x-1}=-2$

e $\dfrac{4}{x-2}+\dfrac{3}{x-1}=3$

f $\dfrac{x}{x-2}-\dfrac{x+3}{x}=-5$

g $\dfrac{1}{x+3}+\dfrac{1}{x-3}=x$

h $\dfrac{4}{x+2}-\dfrac{2}{x+1}=x$

8 Solve for x:

a $x^4-5x^2+4=0$

b $x^4-7x^2+12=0$

c $x^4=4x^2+5$

Hint: Treat these as quadratics in the variable x^2.

9 Solve for x:

a $x^3-10x^2=39x$

b $3x^4-24x^3+48x^2=0$

c $2x^6+88x^2=30x^4$

10 Solve for x:

a $\sqrt{x+2}=x$

b $\sqrt{x+13}-\sqrt{7-x}=2$

1 **a** $x = 0$ or 7
b $x = 0$ or 5
c $x = 0$ or 8
d $x = 0$ or 4
e $x = 0$ or -2
f $x = 0$ or $-\frac{5}{2}$
g $x = 0$ or $\frac{3}{4}$
h $x = 0$ or $\frac{5}{4}$
i $x = 0$ or 3

2 **a** $x = -1$ or -2
b $x = 1$ or 2
c $x = 5$
d $x = -2$ or -3
e $x = 2$ or 3
f $x = -1$ or -6
g $x = -2$ or -7
h $x = -5$ or -6
i $x = -5$ or 3
j $x = -6$ or 2
k $x = 3$ or 8
l $x = 7$

3 **a** $x = -3$ or -6
b $x = -4$ or -7
c $x = -4$ or 2
d $x = -4$ or 3
e $x = 3$ or 2
f $x = 2$
g $x = 3$ or -2
h $x = -12$ or 5
i $x = 10$ or -7
j $x = \frac{1}{2}$ or 2
k $x = -3$ or $\frac{1}{3}$
l $x = -4$ or $-\frac{5}{3}$
m $x = \frac{1}{2}$ or -3
n $x = \frac{1}{2}$ or 5
o $x = -1$ or $-\frac{5}{2}$

4 **a** $x = -\frac{1}{3}$ or -4 **b** $x = -\frac{2}{5}$ or 3 **c** $x = \frac{1}{2}$ or -9
d $x = -1$ or $\frac{5}{2}$ **e** $x = \frac{4}{3}$ or -2 **f** $x = \frac{3}{2}$ or -6
g $x = -\frac{1}{6}$ or 3 **h** $x = \frac{3}{2}$ or -4 **i** $x = \frac{3}{2}$ or $\frac{1}{3}$
j $x = -\frac{8}{3}$ or 2 **k** $x = \frac{4}{7}$ or $-\frac{1}{2}$ **l** $x = \frac{1}{4}$ or $-\frac{4}{3}$
5 **a** $x = -4$ or -3 **b** $x = -3$ or 1 **c** $x = \pm 3$
d $x = -1$ or $\frac{2}{3}$ **e** $x = -\frac{1}{2}$ **f** $x = \frac{5}{2}$ or 4
g $x = 11$ or -3 **h** $x = -\frac{3}{4}$ or $\frac{5}{2}$
6 **a** $x = -2$ or 1 **b** $x = -6$ or 2 **c** $x = 2$ or -1
d $x = 4$ or -1 **e** $x = 1$ or $-\frac{1}{3}$ **f** $x = \frac{1}{2}$ or -1
7 **a** $x = 4$ or -1 **b** $x = 15$ or 2 **c** $x = -9$ or -2
d $x = 3$ or $-\frac{7}{2}$ **e** $x = 4$ or $\frac{4}{3}$ **f** $x = 1$ or $\frac{6}{5}$
g $x = 0$ or $\pm\sqrt{11}$ **h** $x = 0$ or -3
8 **a** $x = \pm 1$ or ± 2 **b** $x = \pm\sqrt{3}$ or ± 2 **c** $x = \pm\sqrt{5}$
9 **a** $x = 0$, 13, or -3 **b** $x = 0$ or 4
c $x = 0$, ± 2, or $\pm\sqrt{11}$
10 **a** $x = 2$ **b** $x = 3$